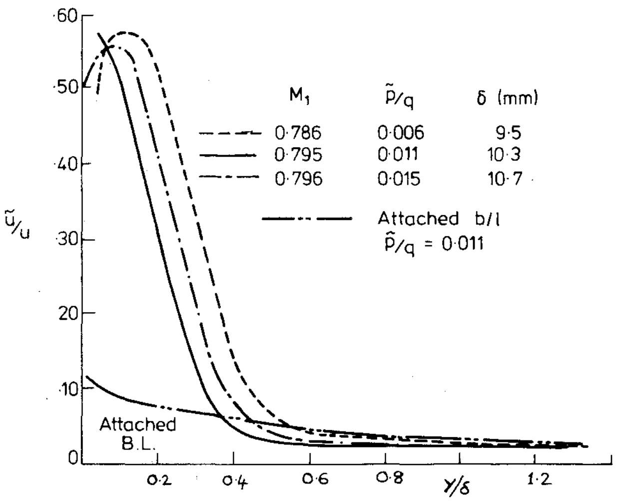

Fig. 5 Turbulence profiles at the trailing edge.

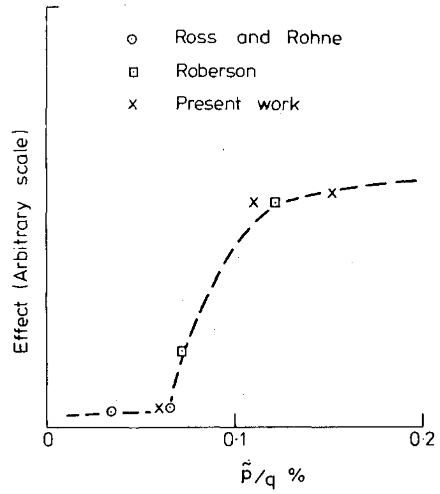

Fig. 6 A model for the effect of pressure fluctuations.

The boundary-layer velocity profiles at the trailing edge of the model for the three levels of pressure fluctuations are shown in Fig. 4. There is a considerable change in the velocity profiles when the levels of $\tilde{p}/q$ were changed from .006 to .011. The changes, with further increases in $\tilde{p}/q$, are not noticeable.

Figure 5 shows the turbulence profiles at the trailing edge for the three levels of $\tilde{p}/q$. A typical turbulence profile obtained in an attached turbulent boundary layer at zero pressure gradient[1] is also shown for comparison. The levels of turbulence in the separated layer are generally an order higher than those in an attached boundary layer. The levels are sensitive to the changes in $\tilde{p}/q$. With the increase in pressure fluctuation levels, the turbulence initially increases and then decreases. However, the changes in the freestream turbulence levels are insignificant for all levels of $\tilde{p}/q$ remaining constant at about 2.4%

The general effect of pressure fluctuations on the shock/boundary-layer interaction is shown schematically in Fig. 6. It is suggested that there is virtually no effect of pressure fluctuations up to a level of about 0.6%. With the increase in pressure fluctuations, whether the effect beyond this level is gradual or sudden is uncertain and needs further investigation.

Acknowledgment

This work originated as part of the research supported by S.R.C. Contract B/RG/8932 and GR/A/53772.

References

[1] Raghunathan, S., Coll, J. B., and Mabey, D. G., "Flat Plate Turbulent Boundary Layers Subject to Large Pressure Fluctuations," *AIAA Journal*, to be published.

[2] Robertson, J. M., "Unsteady Pressure Phenomena for Basic Missile Shapes at Transonic Speeds," AIAA Paper 64-3, 1974.

[3] Ross, R. and Rohne, P. B., "The Character of Flow Unsteadiness and Its Influence on Steady State Transonic Wind Tunnel Measurements," AGARD CP-174, Paper 45.

[4] Mabey, D. G., "Flow Unsteadiness of Mode Vibration in Wind Tunnels at Subsonic or Transonic Speeds," ARC CP 1155, 1971.

Radiation Shape Factors between End Plane and Outer Wall of Concentric Tubular Enclosures

C. P. Minning*

Hughes Aircraft Company, Culver City, Calif.

Nomenclature

x,y,z	= Cartesian coordinates of dA_1, cm
l_1,m_1,n_1	= cosines (i.e., direction cosines) of the angles between the normal to dA_1 and the x, y, and z axes, respectively
x_2,y_2,z_2	= Cartesian coordinates of a point on the periphery of surface 2, cm
S	= distance between dA_1 and a point on the periphery of surface 2, cm
r	= radial coordinate in plane of dA_1, cm
r_o	= radius of outer tube, cm
r_i	= radius of inner tube, cm
θ	= angular displacement from x axis, rad
ω	= view angle, rad
h	= height of concentric tubes, cm

Introduction

IN radiation heat-transfer calculations for diverse applications such as gas turbine combustion chambers, rotary-kiln dryers, and spin-stabilized spacecraft, it is often desired to calculate the shape factor between an annular-disk end plane and the outer wall of an enclosure formed by concentric cylinders, such as that shown in Fig. 1. If the inner and outer radii of the disk are the same as the radii of the inner and outer cylinders, the shape factor can be calculated by means of the closed-form expressions given in Ref. 2. However, there is no closed-form expression available in the literature for the case where the radii of the annular disk are not equal to the radii of the concentric cylinders.

The contour integral method is used in the present analysis to derive a closed-form expression for the shape factor, $F_{dA_1\text{-}A_2}$, from a differential area on the annular-disk end plane to the outer wall of the enclosure. This expression can then be integrated numerically to obtain the desired shape factor from a finite-sized annular disk to the outer wall. Typical results are shown for both configurations.

Analysis

Consider the geometry shown in Figs. 1-3 which illustrate the nomenclature used in this presentation. Throughout this

Received May 10, 1979.

Index categories: Spacecraft Temperature Control; Radiation and Radiative Heat Transfer; Thermal Modeling and Analysis.

*Senior Staff Engineer, Electro-Optical and Data Systems Group.

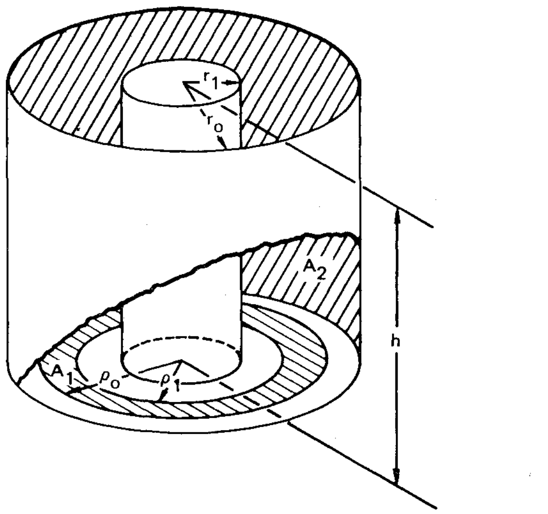

Fig. 1 Nomenclature for shape factors between a finite-sized annular disk and the outer wall of a concentric cylindrical enclosure.

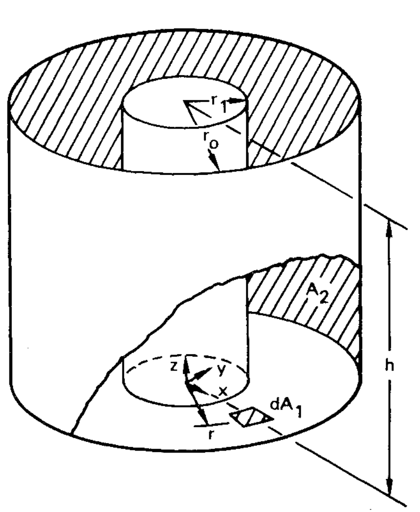

Fig. 2 Nomenclature for shape factor from a differential area to the outer wall of a concentric cylindrical enclosure.

discussion, surface 2 is always considered to be of finite size; surface 1 can be either a differential area or a finite-sized area depending on the problem under consideration.

Sparrow[3] has shown that $F_{dA_1\text{-}A_2}$ can be expressed as the sum of three contour integrals in the following manner:

$$F_{dA_1\text{-}A_2}=l_1\oint_C \frac{(z_2-z)\,dy_2-(y_2-y)\,dz_2}{2\pi S^2}$$

$$+m_1\oint_C \frac{(x_2-x)\,dz_2-(z_2-z)\,dx_2}{2\pi S^2}$$

$$+n_1\oint_C \frac{(y_2-y)\,dx_2-(x_2-x)\,dy_2}{2\pi S^2} \tag{1}$$

where

$$S^2=(x_2-x)^2+(y_2-y)^2+(z_2-z)^2 \tag{2}$$

Here, C designates integration around the periphery of surface 2. Since the normal to dA_1 is parallel to the z axis, $l_1=m_1=0$, and $n_1=1$. Hence, only the last term in Eq. (1) is nonzero.

Integration around the periphery of surface 2 is performed in the following sequence: a) along boundary 1, where $\theta=-\omega$

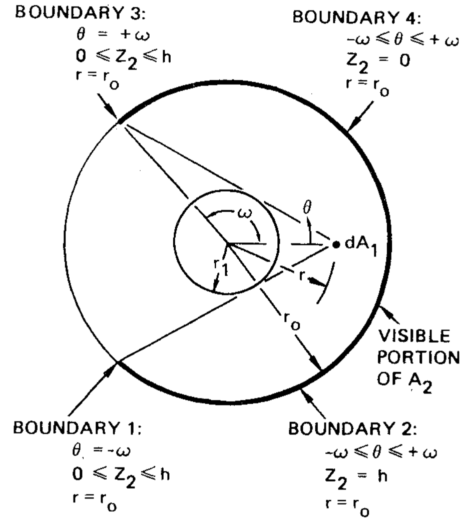

Fig. 3 Visible portion of outer wall of a concentric cylindrical enclosure as seen from the differential area, dA_1.

and z_2 is increased from 0 to h; b) along boundary 2, where $z_2=h$ and θ is increased from $-\omega$ to $+\omega$; c) along boundary 3, where $\theta=+\omega$ and z_2 is decreased from h to 0; and d) along boundary 4, where $z_2=0$ and θ is decreased from $+\omega$ to $-\omega$. Now both dx_2 and dy_2 are equal to zero along boundaries 1 and 3; therefore, the only nonzero integrals in Eq. (1) correspond to boundaries 2 and 4. For both boundaries 2 and 4, $x_2=r_o\cos\theta$, $dx_2=-r_o\sin\theta d\theta$, $y_2=r_o\sin\theta$, $dy_2=r_o\cos\theta d\theta$, $x=r$, $y=0$, and $z=0$. For boundary 2, $z_2=h$, and for boundary 4, $z_2=0$. Only that portion of A_2 which is located in the region $-\omega\leq\theta\leq\omega$ is visible at dA_1. The view angle ω is defined below (see Fig. 3).

$$\omega=\cos^{-1}(r_1/r)+\cos^{-1}(r_1/r_o) \tag{3}$$

Substitution of the appropriate expressions for x,y,z,x_2,y_2, and z_2 into Eqs. (1) and (2) results in the following expressions:

$$F_{dA_1\text{-}A_2}=\frac{-r_o}{2\pi}\int_{-\omega}^{\omega}\frac{(r_o-r\cos\theta)\,d\theta}{r_o^2-2rr_o\cos\theta+r^2+h^2}$$

$$-\frac{r_o}{2\pi}\int_{\omega}^{-\omega}\frac{(r_o-r\cos\theta)\,d\theta}{r_o^2-2rr_o\cos\theta+r^2} \tag{4}$$

The integrals in this expression are available in standard references.[1] The desired expression for $F_{dA_1\text{-}A_2}$ is then found to be

$$F_{dA_1\text{-}A_2}=\frac{1}{\pi}\tan^{-1}\left[\frac{r_o+r}{r_o-r}\tan(\omega/2)\right]$$

$$-\frac{(r_o^2-r^2-h^2)}{\pi\sqrt{(r_o^2+r^2+h^2)^2-(4r^2r_o^2)}}$$

$$\times\tan^{-1}\left[\frac{\sqrt{(r_o^2+r^2+h^2)^2-4(r^2r_o^2)}}{r_o^2+r^2+h^2-2rr_o}\tan(\omega/2)\right] \tag{5}$$

Now consider the special case where $r_1=0$. For this situation, $\omega=\pi$ and Eq. (5) reduces to

$$F_{dA_1\text{-}A_2}=\frac{1}{2}\left[1-\frac{r_o^2-r^2-h^2}{\sqrt{(r_o^2+r^2+h^2)^2-4(r^2r_o^2)}}\right] \tag{6}$$

which is the same answer obtained when the familiar expression for the shape factor from an off-center differential

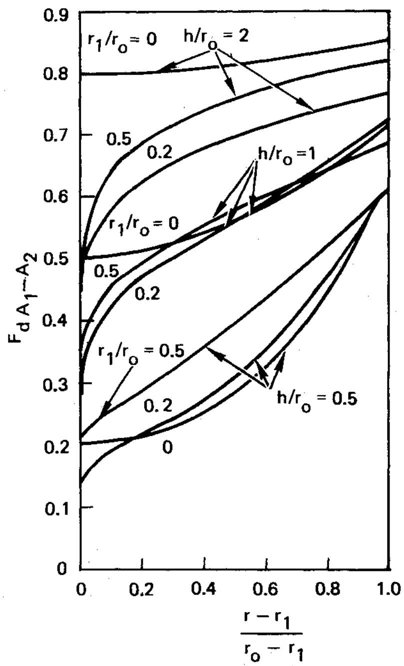

Fig. 4 Shape factor, $F_{dA_I\text{-}A_2}$, from a differential area to the outer wall of a concentric cylindrical enclosure as a function of radial distance parameter, $(r-r_I)/(r_o-r_I)$.

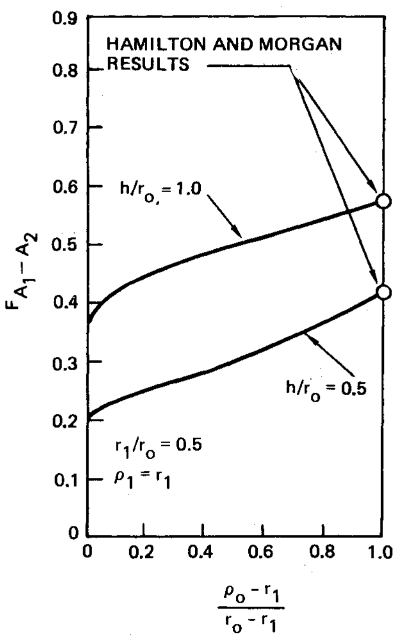

Fig. 5 Shape factor, $F_{dA_I\text{-}A_2}$, from a finite-sized annular disk to the outer wall of a concentric cylindrical enclosure as a function of radial distance parameter $(\rho_0-r_I)/(r_0-r_I)$.

area to a circular disk

$$F_{dA_I\text{-disk}} = \frac{1}{2}\left[1 + \frac{r_o^2 - r^2 - h^2}{\sqrt{(r_o^2 + r^2 + h^2)^2 - 4(r^2 r_o^2)}}\right] \tag{7}$$

is subtracted from one.

Shape factors between a differential area and the outer wall of a concentric cylindrical enclosure are plotted against the quantity $(r-r_I)/(r_o-r_I)$, for several values of r_I/r_o and h/r_o in Fig. 4.

Shape factors between a finite-sized annular disk and the outer wall of a concentric cylindrical enclosure can be determined by integrating Eq. (5) in the following manner:

$$F_{A_I\text{-}A_2} = \frac{2}{(\rho_0^2 - \rho_I^2)} \int_{\rho_I}^{\rho_0} r F_{dA_I\text{-}A_2} \mathrm{d}r \tag{8}$$

The integral in this expression can be evaluated numerically using standard techniques such as Simpson's Rule. Typical results are shown in Fig. 5. Note that for $\rho_I = r_I$ and $\rho_0 = r_o$, the results are identical to those given for configurations A7 and A8 in Ref. 2.

References

[1] Gradshteyn, I. S. and Ryzhik, I. M., *Table of Integrals, Series, and Products*, 4th ed., trans. and ed. by Alan Jeffrey, Academic Press, New York, 1965, p. 148.

[2] Hamilton, D. C. and Morgan, W. R., "Radiant-Interchange Configuration Factors," NACA TN 2836, 1952, pp. 37 and 42.

[3] Sparrow, E. M., "A New and Simpler Formulation for Radiative Angle Factors," *Journal of Heat Transfer, ASME Transactions, Series C*, Vol. 85, Feb. 1963, pp. 81-88.

U.S. POSTAL SERVICE

STATEMENT OF OWNERSHIP, MANAGEMENT AND CIRCULATION

(Required by 39 U.S.C. 3685)

1. TITLE OF PUBLICATION: AIAA JOURNAL	A. PUBLICATION NO.: 0 0 9 9 2 0	2. DATE OF FILING: OCTOBER 1, 1979
3. FREQUENCY OF ISSUE: MONTHLY	A. NO. OF ISSUES PUBLISHED ANNUALLY: 12	B. ANNUAL SUBSCRIPTION PRICE: $10.00

4. LOCATION OF KNOWN OFFICE OF PUBLICATION *(Street, City, County, State and ZIP Code) (Not printers)*
1290 AVENUE OF THE AMERICAS, NEW YORK, N.Y. 10019

5. LOCATION OF THE HEADQUARTERS OR GENERAL BUSINESS OFFICES OF THE PUBLISHERS *(Not printers)*
SAME AS ABOVE

6. NAMES AND COMPLETE ADDRESSES OF PUBLISHER, EDITOR, AND MANAGING EDITOR

PUBLISHER *(Name and Address)* AMERICAN INSTITUTE OF AERONAUTICS AND ASTRONAUTICS, INC.
1290 AVENUE OF THE AMERICAS, NEW YORK, N.Y. 10019

EDITOR *(Name and Address)*
GEORGE W. SUTTON SAME AS ABOVE

MANAGING EDITOR *(Name and Address)*
ELAINE J. CAMHI SAME AS ABOVE

7. OWNER *(If owned by a corporation, its name and address must be stated and also immediately thereunder the names and addresses of stockholders owning or holding 1 percent or more of total amount of stock. If not owned by a corporation, the names and addresses of the individual owners must be given. If owned by a partnership or other unincorporated firm, its name and address, as well as that of each individual must be giving. If the publication is published by a nonprofit organization, its name and address must be stated.)*

NAME	ADDRESS
AMERICAN INSTITUTE OF AERONAUTICS AND ASTRONAUTICS, INC.	SAME AS ABOVE

8. KNOWN BONDHOLDERS, MORTGAGEES, AND OTHER SECURITY HOLDERS OWNING OR HOLDING 1 PERCENT OR MORE OF TOTAL AMOUNT OF BONDS, MORTGAGES OR OTHER SECURITIES *(If there are none, so state)*

NAME	ADDRESS
NONE	

9. FOR COMPLETION BY NONPROFIT ORGANIZATIONS AUTHORIZED TO MAIL AT SPECIAL RATES *(Section 132.122, PSM)*
The purpose, function, and nonprofit status of this organization and the exempt status for Federal income tax purposes *(Check one)*

[XX] HAVE NOT CHANGED DURING PRECEDING 12 MONTHS [ ] HAVE CHANGED DURING PRECEDING 12 MONTHS *(If changed, publisher must submit explanation of change with this statement.)*

10. EXTENT AND NATURE OF CIRCULATION	AVERAGE NO. COPIES EACH ISSUE DURING PRECEDING 12 MONTHS	ACTUAL NO. COPIES OF SINGLE ISSUE PUBLISHED NEAREST TO FILING DATE
A. TOTAL NO. COPIES PRINTED *(Net Press Run)*	6,300	6,400
B. PAID CIRCULATION 1. SALES THROUGH DEALERS AND CARRIERS, STREET VENDORS AND COUNTER SALES	-----	-----
2. MAIL SUBSCRIPTIONS	5,376	5,490
C. TOTAL PAID CIRCULATION *(Sum of 10B1 and 10B2)*	5,376	5,490
D. FREE DISTRIBUTION BY MAIL, CARRIER OR OTHER MEANS SAMPLES, COMPLIMENTARY, AND OTHER FREE COPIES	-----	-----
E. TOTAL DISTRIBUTION *(Sum of C and D)*	5,376	5,490
F. COPIES NOT DISTRIBUTED 1. OFFICE USE, LEFT OVER, UNACCOUNTED, SPOILED AFTER PRINTING	924	910
2. RETURNS FROM NEWS AGENTS	-----	-----
G. TOTAL *(Sum of E, F1 and 2—should equal net press run shown in A)*	6,300	6,400

11. I certify that the statements made by me above are correct and complete. SIGNATURE AND TITLE OF EDITOR, PUBLISHER, BUSINESS MANAGER, OR OWNER
Sonia Valle
SONIA VALLE, MANAGER, ADMINISTRATIVE SERVICES

12. FOR COMPLETION BY PUBLISHERS MAILING AT THE REGULAR RATES *(Section 132.121, Postal Service Manual)*

39 U. S. C. 3626 provides in pertinent part: "No person who would have been entitled to mail matter under former section 4359 of this title shall mail such matter at the rates provided under this subsection unless he files annually with the Postal Service a written request for permission to mail matter at such rates."

In accordance with the provisions of this statute, I hereby request permission to mail the publication named in Item 1 at the phased postage rates presently authorized by 39 U. S. C. 3626.

SIGNATURE AND TITLE OF EDITOR, PUBLISHER, BUSINESS MANAGER, OR OWNER
SONIA VALLE, MANAGER, ADMINISTRATIVE SERVICES Sonia Valle